10/8/19

MIS6 (Continued)

Case 3: Incomplete Individual Data (Continued)

Confidence Intervals (Two Types)

1) The "95%" linear symmetric confidence interval based on the normal approximation for a parameter is:

$$\text{estimate} \pm \text{"}1.96\text{"} \sqrt{V}$$

where V = approximate variance of the estimator

2) (a) The "95%" log-transformed confidence interval for $S_X(t)$ using Kaplan-Meier is:

$$\left([S_n(t)]^{1/\alpha}, [S_n(t)]^{\alpha} \right)$$

where $\alpha = e^{\frac{\text{"}1.96\text{"} \cdot \sqrt{V}}{S_n(t) \cdot \ln(S_n(t))}}$ and V = approximate variance of KM estimator

(b) The "95%" log-transformed confidence interval for $H_X(t)$ using Nelson-Aalen is:

$$\left(\frac{1}{\beta} \cdot \hat{H}(t), \beta \cdot \hat{H}(t) \right)$$

where $\beta = e^{\frac{\text{"}1.96\text{"} \sqrt{V}}{\hat{H}(t)}}$ and V = approximate variance of NA estimator for $H_X(t)$

Examples (See Next Pages)

1) (LM.4. / SOA Sample Questions)

2) (#6 / WA Spring 2019)

LM.4. You are given the following data based on 60 observations:

i	y_i	s_i	b_i	r_i
1	5	5	7	60
2	8	6	7	48
3	13	7	7	35
4	16	6	5	21
5	21	6	4	10

Calculate the upper limit of the 80% confidence interval for $S(21)$ using the Kaplan-Meier estimate and Greenwood's approximation.

(A) 0.249

(B) 0.283

(C) 0.311

(D) 0.335

(E) 0.351

Note: The 90^{th} percentile of the SND is 1.282

$\therefore$ Answer $= \underbrace{\text{estimate}}_{S_{60}(21)} + 1.282 \cdot \sqrt{V_{60}}$

$$\therefore S_{60}(21) = \prod_{y_j \le 21}\left(1 - \frac{s_j}{r_j}\right) = \left(1-\frac{5}{60}\right)\left(1-\frac{6}{48}\right)\left(1-\frac{7}{35}\right)\left(1-\frac{6}{21}\right)\left(1-\frac{6}{10}\right)$$

$$= 0.18\overline{3}$$

$$V_{60} = [S_{60}(21)]^2 \cdot \sum_{y_j \le 21} \frac{s_j}{r_j(r_j - s_j)}$$

$$= [0.18\overline{3}]^2 \cdot \left(\frac{5}{60 \cdot 55} + \frac{6}{48 \cdot 42} + \frac{7}{35 \cdot 28} + \frac{6}{21 \cdot 15} + \frac{6}{10 \cdot 4}\right)$$

Answer $= 0.18\overline{3} + 1.282\,(0.07792\ldots) = 0.28323\ldots$

6. (*10 points*) In a mortality study of a cohort of twelve 80-year-olds, you are given the following observed exit times:

$$1^+,\ 2,\ 2,\ 2^+,\ 4,\ 4^+,\ 5,\ 5^+,\ 7,\ 8,\ 9^+,\ 9^+$$

where "+" indicates censoring; all other exits are deaths.

Let $\hat{S}(t)$ denote the Kaplan-Meier estimate of the time t survival probability, $S(t)$, for a life age 80.

(a) (*3 points*) Specify $\hat{S}(t)$ for all values of $t, 0 < t \le 9$, and hence show that $\hat{S}(6) = 0.60$ to the nearest 0.01. You should calculate $\hat{S}(6)$ to the nearest 0.001.

(b) (*2 points*) Show that the estimated standard deviation of $\hat{S}(6)$ using Greenwood's formula is 0.16 to the nearest 0.01. You should calculate your value to the nearest 0.001.

(c) (*3 points)*

(i) Calculate an approximate 95% linear confidence interval for $S(6)$.

(ii) Calculate an approximate 95% log-transformed confidence interval for $S(6)$.

(d) (*1 point*) Explain why the log-transformed confidence interval is preferred when estimating $S(t)$.

(e) (*1 point*) Suppose that you have a thirteenth observation truncated at time 3 and right-censored before time 4. Without further calculation explain whether this would increase, decrease or have no effect on your estimate of $S(6)$.

****END OF EXAMINATION****

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y_j	s_j	r_j
2	2	11
4	1	8
5	1	6
7	1	4
8	1	3

$$S_{12}(t) = \prod_{y_j \le t} \left(1 - \frac{s_j}{r_j}\right)$$

(a)

For $0 \le t < 2$, $S_{12}(t) = 1$

For $2 \le t < 4$, $S_{12}(t) = \left(1 - \frac{2}{11}\right)$

For $4 \le t < 5$, $S_{12}(t) = \left(1 - \frac{2}{11}\right)\left(1 - \frac{1}{8}\right)$

For $5 \le t < 7$, $S_{12}(t) = \left(1 - \frac{2}{11}\right)\left(1 - \frac{1}{8}\right)\left(1 - \frac{1}{6}\right) = \frac{9}{11} \cdot \frac{7}{8} \cdot \frac{5}{6} = 0.597$

For $7 \le t < 8$, $S_{12}(t) = \frac{9}{11} \cdot \frac{7}{8} \cdot \frac{5}{6} \cdot \left(1 - \frac{1}{4}\right)$

~~For~~ $S_{12}(8) = \frac{9}{11} \cdot \frac{7}{8} \cdot \frac{5}{6} \cdot \frac{3}{4} \cdot \frac{2}{3}$

(b) $$\sqrt{V_{12}} = \sqrt{[S_{12}(6)]^2 \cdot \sum_{y_j \le 6} \frac{s_j}{r_j(r_j - s_j)}}$$

$$= \sqrt{(0.597)^2 \cdot \left(\frac{2}{11 \cdot 9} + \frac{1}{8 \cdot 7} + \frac{1}{6 \cdot 5}\right)} = 0.159$$

(c) (i) $S_{12}(6) \pm 1.96\sqrt{V_{12}}$

$0.597 \pm 1.96\,(0.159)$

$(0.28536,\ 0.90864)$

(ii) Note: $\alpha = e^{\frac{1.96\,(0.159)}{0.597 \cdot \ln(0.597)}} = 0.3635\ldots$

$\left((0.597)^{1/0.3635},\ (0.597)^{0.3635}\right)$